1. MEASURING INEQUALITY

Properties (axioms) of inequality metrics

$\boldsymbol{y}$ = vector of indicators to be analyzed, $\boldsymbol{y} = (y_1, \ldots, y_i, \ldots, y_n)$
$I(\boldsymbol{y})$ = an inequality measure

- **Anonymity (symmetry):**
  if $\boldsymbol{y^*}$ is a permutation of $\boldsymbol{y}$, then $I(\boldsymbol{y^*}) = I(\boldsymbol{y})$
- **Scale invariance:**
  $I(\lambda \boldsymbol{y}) = I(\boldsymbol{y})$
- **Translation invariance:**
  if $\boldsymbol{y^*} = (y_1+\theta, \ldots, y_i+\theta, \ldots, y_n+\theta)$, then $I(\boldsymbol{y^*}) = I(\boldsymbol{y})$
- **Population independence:**
  if $\boldsymbol{y^*} = \boldsymbol{y} \oplus \mathbf{y} \oplus \ldots \oplus \boldsymbol{y}$, then $I(\boldsymbol{y^*}) = I(\boldsymbol{y})$
- **Transfer principle:**
  if $\boldsymbol{y^*} = (y_1, \ldots, y_i+\delta, \ldots, y_k-\delta, \ldots, y_n)$ and $y_i+\delta < y_k-\delta$, then $I(\boldsymbol{y^*}) < I(\boldsymbol{y})$
- **Decomposability**
  if $\boldsymbol{y} = \boldsymbol{y}_{(1)} \oplus \mathbf{y}_{(2)} \oplus \ldots \oplus \boldsymbol{y}_{(m)}$, then $I(\boldsymbol{y}) = I(\boldsymbol{y}_{(1)}) + I(\boldsymbol{y}_{(2)}) + \ldots + I(\boldsymbol{y}_{(m)})$

INEQUALITY INDICATORS

Standard deviation of logarithms: $$\sigma = \frac{1}{N}\sum_{i=1}^{N}(\ln y_i - (\overline{\ln y})^2)$$

Gini index: $$G = \frac{1}{2N^2\bar{y}}\sum_{i=1}^{N}\sum_{j=1}^{N}\left|y_i - y_j\right| = \frac{\sum_{i=1}^{N}(2r_i - N - 1)y_i}{N\sum_{i=1}^{N}y_i} = 2\frac{\operatorname{cov}(y,r)}{N\bar{y}}$$

Generalized entropy index: $$GE(\alpha) = \frac{1}{\alpha^2 - \alpha}\left(\frac{1}{N}\sum_{i=1}^{N}\left(\frac{y_i}{\bar{y}}\right)^{\alpha} - 1\right)$$

α=0: mean log deviation $$GE(0) = \frac{1}{N}\sum_{i=1}^{N} \ln \frac{\bar{y}}{y_i}$$

α=1: Theil index $$GE(1) = \frac{1}{N}\sum_{i=1}^{N} \frac{y_i}{\bar{y}} \ln \frac{y_i}{\bar{y}}$$

α=2: coefficient of variation: $$GE(2) = \frac{1}{\bar{y}}\left[\frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{y})^2\right]^{\frac{1}{2}}$$

Atkinson index: $$A(\varepsilon) = 1 - \frac{1}{\bar{y}}\left(\frac{1}{N}\sum_{i=1}^{N} y_i^{1-\varepsilon}\right)^{1/(1-\varepsilon)}, \varepsilon \neq 1$$

$$A(1) = 1 - \frac{1}{\bar{y}}\left(\prod_{i=`1}^{N} y_i\right)^{1/N}$$