# 2. THE ECONOMIC GROWTH THEORY AND INCOME CONVERGENCE

# NEOCLASSICAL MODEL OF ECONOMIC GROWTH

- Production function: $Y(t) = K(t)^a(L(t)A(t))^{1-a} = K(t)^a \tilde{L}(t)^{1-a}$
  $\partial K(t)/\partial t = sY(t) - \delta K(t)$, $L(t) = L(0)e^{\nu t}$, $A(t) = A(0)e^{\xi t}$
- Income per «effective worker» : $\hat{y}(t) = Y(t)/L(t)A(t) = \tilde{k}(t)^a$
  income per capita: $y(t) = Y(t)/L(t) = \hat{y}(t)A(0)e^{\xi t}$
- Equilibrium growth: $\tilde{k}(t) = \tilde{k}^* \Rightarrow s \cdot \hat{y}^* = (\xi + \nu + \delta)\tilde{k}^*$, $\hat{y}^* = f(\tilde{k}^*)$,
  hence $\ln\hat{y}(t) - \ln\hat{y}^* = (\ln\hat{y}(0) - \ln\hat{y}^*)e^{-\lambda t}$, $\lambda>0$: $\hat{y}(t) \to \hat{y}^*$ with $t \to \infty$
- In terms of per capita income:
  $\ln y(t) =$ $\ln\hat{y}^* + \ln A(0) + \xi t$ + $(\ln y(0) - \ln A(0) - \ln\hat{y}^*)e^{-\lambda t}$
  equilibrium growth path — deviation from the equilibrium growth path
- $\beta$-convergence:
  $\ln y(t) = \underline{(\ln A(0) + \ln\hat{y}^*)(1 - e^{-\lambda t}) + \xi t} + \underline{e^{-\lambda t}}\ln y(0)$
  $\ln y(t) = \alpha + \beta_+ \ln y(0)$, $\beta_+ < 1$
  or $\ln(y(t)/y(0)) = \alpha + \beta \ln y(0)$, $\beta = \beta_+ - 1 < 0$

**Notation:**

$Y$ – total output; $s$ – saving rate;
$K$ – physical capital; $\delta$ – depreciation rate of physical capital;
$L$ – number of workers; $\tilde{k}$ – capital per «effective worker»;
$A$ – state of technology; $\lambda$ – rate of convergence to the equilibrium.

# TWO TYPES OF CONVERGENCE
## in set $\{i\}$ of economies

- **Unconditional convergence**

economies are homogeneous: $f_i(\cdot)$, $\xi_i$ , $\nu_i$ , $\delta_i$ , $s_i$ , $A_i(0)$ are the same for all $i$, then $\hat{y}_i^* = g((\xi_i + \nu_i + \delta_i)/s_i)$ and economic growth path are the same;

$$\ln(y_i(t)/y_i(0)) = \alpha + \beta \ln y_i(0)$$

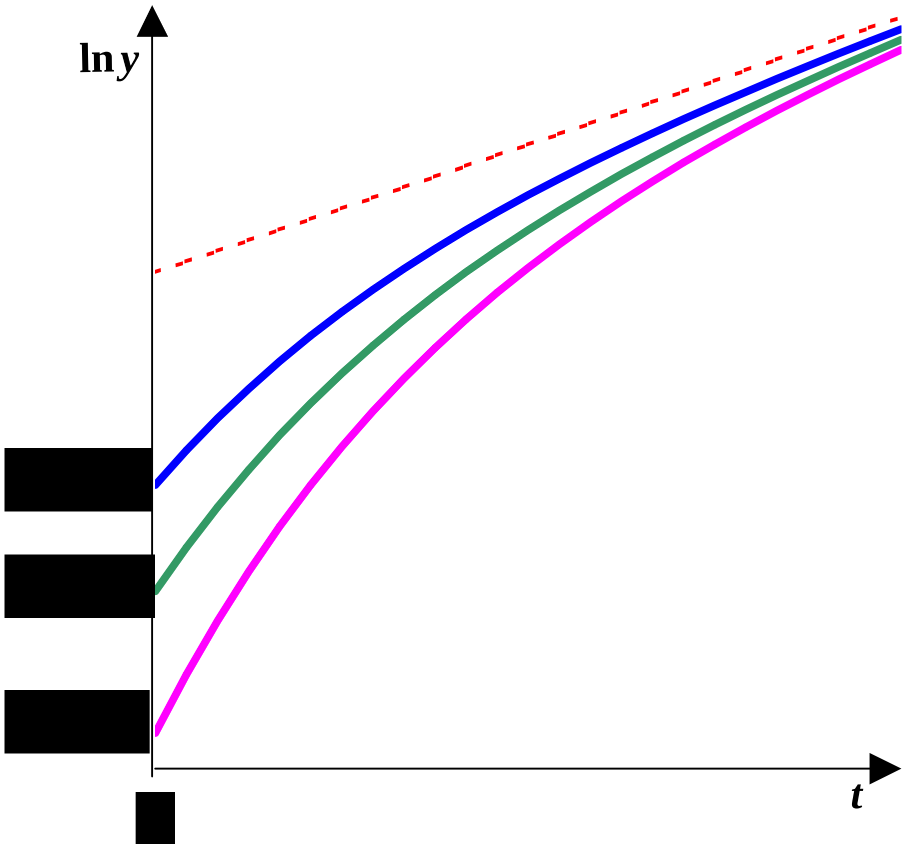
- **Conditional convergence**

economies are heterogeneous, $\alpha = \alpha(f_i(\cdot), \xi_i , \nu_i , \delta_i , s_i , A_i(0))$;

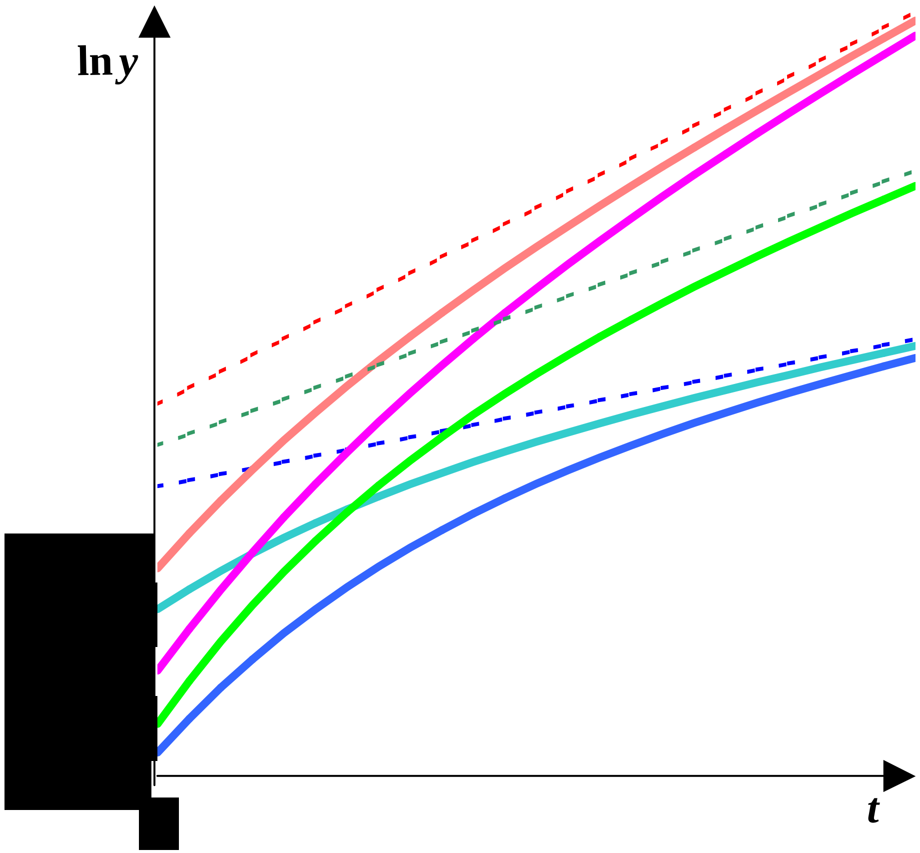
$$\ln(y_i(t)/y_i(0)) = \alpha_0 + \alpha(x_{i1},\ldots, x_{im}) + \beta \ln y_i(0)$$

or $\ln(y^\circ_i(t)/y_i(0)) = \alpha_0 + \beta \ln y_i(0)$,

or $\ln(y^\circ_i(t)/y_i(0)) = \ln(y_i(t)/\ln y_i(0)) - \alpha(x_{i1},\ldots, x_{im})$

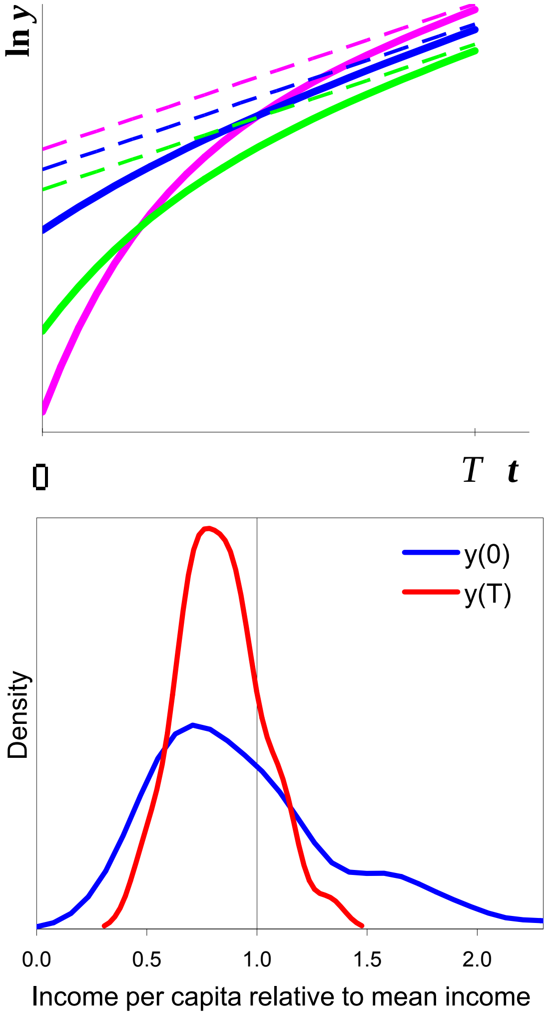



**Unconditional convergence**

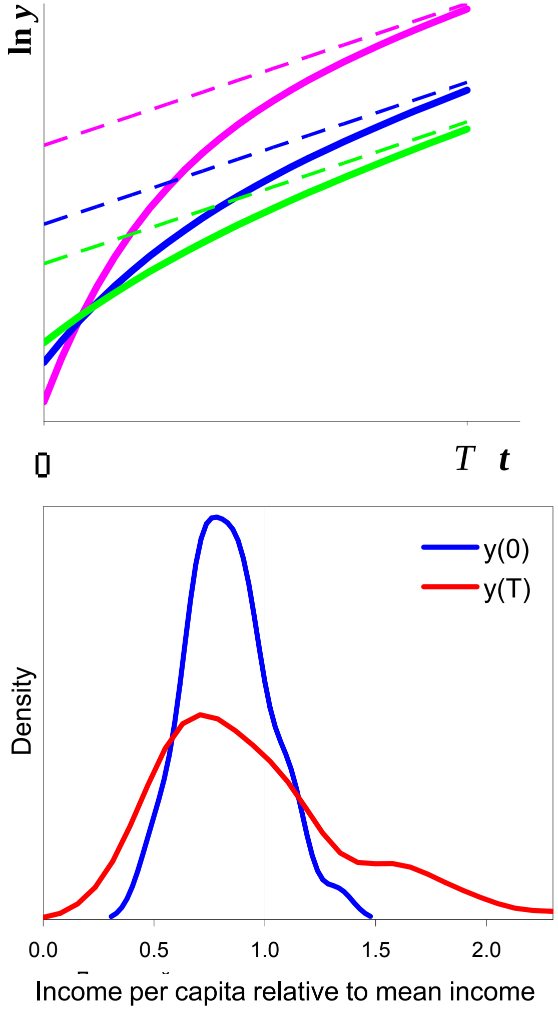



**Conditional convergence**

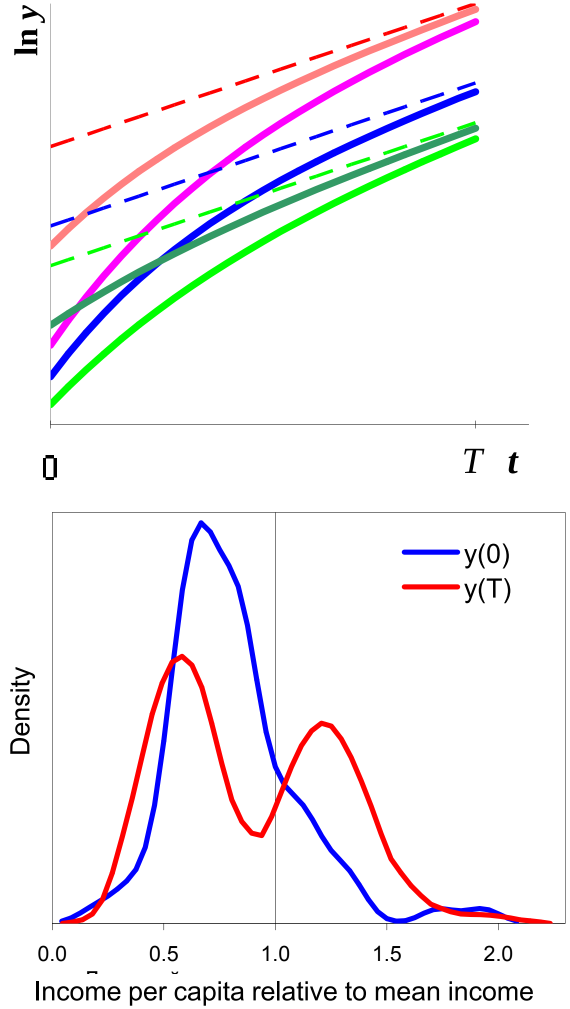
# PREDICTIONS OF THE GROWTH THEORY




Global income convergence




Income divergence




Local (club) convergence

# ALTERNATIVE GROWTH MODELS

- **Two saving rates**: the saving rate out of wage income is $s_1$, the saving rate out of interest income is $s_2$. Then the economy may have two stable equilibriums $\hat{y}^*$. In a set of *homogeneous economies* those with low $y_i(0)$ converge to one equilibrium growth path, and those with high $y_i(0)$ converge to other equilibrium growth path (which implies a “poverty trap”).
- **The Romer model**: increasing marginal productivity. Economies need not converge; growth may be persistently slower in less developed countries or even may fail to take place at all.
- **The Azariadis-Drazen model**: threshold effects. Multiple equilibriums $\hat{y}^*$ in an economy; the evolution differs from conditional convergence.
- And so on.