3. CROSS-SECTION APPROACH

3.1. Beta-convergence
(regression to the mean)

β-CONVERGENCE REGRESSIONS

Two points in time: $t = 0$, $t = T$

- **Unconditional β-convergence:**

$\ln y_{iT} = \alpha + \beta_{+} \ln y_{i0} + \varepsilon_i$, $\quad H_0: \beta_{+} < 1$

$\ln(y_{iT}/y_{i0}) = \alpha + \beta \ln y_{i0} + \varepsilon_i$, $\quad H_0: \beta < 0$

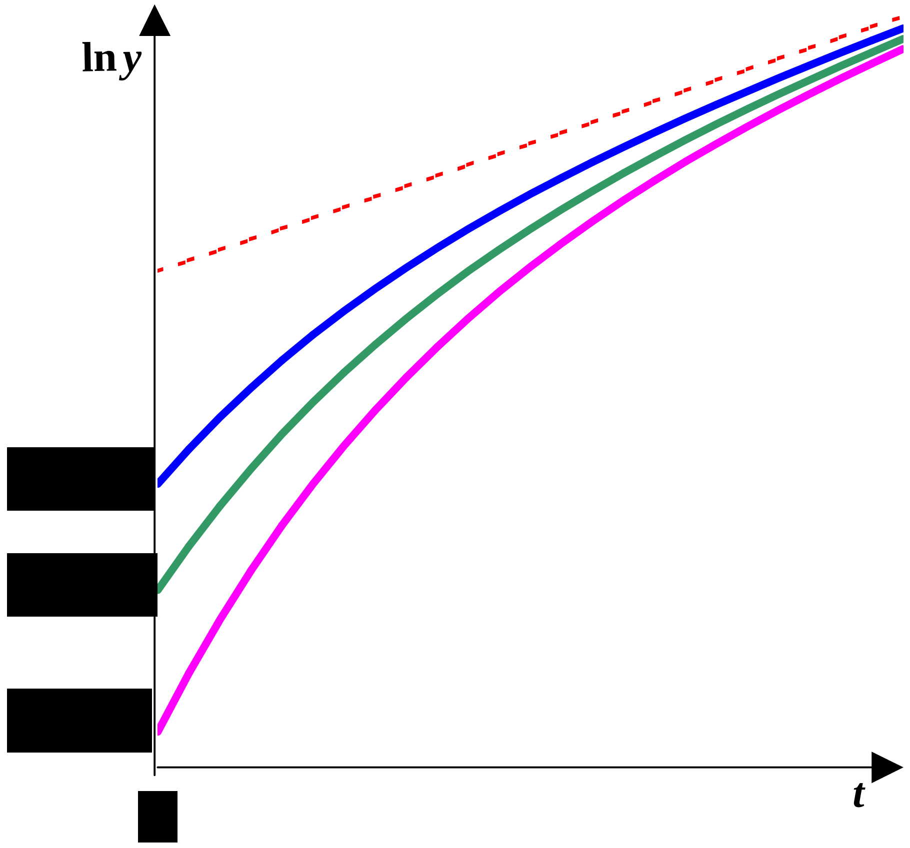
$\ln(y_{iT}/y_{i0})/T = \alpha + \beta \ln y_{i0} + \varepsilon_i$, $\quad H_0: \beta < 0$

- **Conditional β-convergence:**

$\ln y_{iT} = \alpha_0 + \beta_{+} \ln y_{i0} + \sum \alpha_k \ln x_{ik} + \varepsilon_i$, $\quad H_0: \beta_{+} < 1$

$\ln(y_{iT}/y_{i0}) = \alpha_0 + \beta \ln y_{i0} + \sum \alpha_k \ln x_{ik} + \varepsilon_i$, $\quad H_0: \beta < 0$

$\ln(y_{iT}/y_{i0})/T = \alpha_0 + \beta \ln y_{i0} + \sum \alpha_k \ln x_{ik} + \varepsilon_i$, $\quad H_0: \beta < 0$



Unconditional convergence

β-CONVERGENCE vs. σ-CONVERGENCE

Income inequality measure $\sigma_t = \sigma(\ln y_t)$,

σ-convergence: $\sigma_{t+\tau} < \sigma_t$

$\ln(y_{iT}/y_{i0}) = \alpha + \beta \ln y_{i0} + \varepsilon_i$, $\beta < 0$

$$\beta = \frac{\mathrm{cov}(\ln y_0, \ln y_T)}{\sigma_0^2} - 1$$

From $\sigma^2(\ln y_T - \ln y_0) = \sigma_0^2 + \sigma_T^2 - 2\mathrm{cov}(\ln y_0, \ln y_T)$ we get

$$\beta = \frac{1}{2}\left(\left(\frac{\sigma_T^2}{\sigma_0^2} - 1\right) - \frac{\sigma^2(\ln(y_T/y_0))}{\sigma_0^2}\right)$$

- **σ-convergence suggests β-convergence:**
  $\sigma_T < \sigma_0 \Rightarrow \beta < 0$
- **β-convergence does not suggest σ-convergence:**
  $\sigma_T > \sigma_0$ and $\mathrm{cov}(\ln y_0, \ln y_T) < \sigma_0^2 \Rightarrow \beta < 0$
  (e.g., with $\sigma_T = \sigma_0$, we always have $\beta < 0$)

"GALTON'S FALLACY"
and bidirectional β-convergence

$\sigma_T = \sigma_0 \Rightarrow \beta < 0 \; (\beta_+ < 1)$

$\{y_{i0}\}$ = nominal incomes per capita across Russian regions for 2005

$\sigma(\ln y_0) = 0{,}407$; $G(y_0) = 0{,}237$

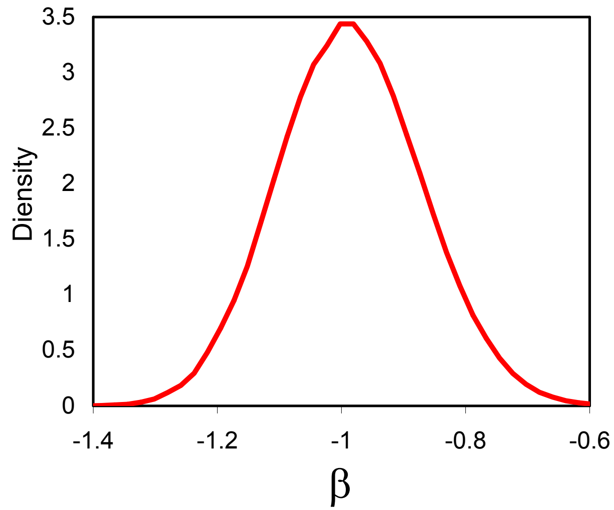
$\{y_{iT}\}$ = permutation of $\{y_{i0}\}$: $y_{T,2k-1} = y_{0,2k}$, $y_{T,2k} = y_{0,2k-1}$, $k = 1,\ldots,39$

$\sigma(\ln y_T) = 0{,}407$; $G(y_T) = 0{,}237$

Regression	Constant α		β	
	Estimate	p-value	Estimate	p-value
$0 \to T$	5,992 (0,945)	0,000	**–0,687** (0,108)	0,000
$T \to 0$	5,992 (0,945)	0,000	**–0,687** (0,108)	0,000

$0 \to T$: $\ln(y_{iT}/y_{i0}) = \alpha + \beta \ln y_{i0} + \varepsilon_i$

$T \to 0$: $\ln(y_{i0}/y_{iT}) = \alpha + \beta \ln y_{iT} + \varepsilon_i$



Random permutations
(100 thousand replications)

β-CONVERGECE WITH σ-DIVERGENCE
An example

$\sigma_T > \sigma_0 \Rightarrow \beta < 0$ ($\beta_+ < 1$)

$\{y_{i0}\}$ = nominal incomes per capita across Russian regions for 2005

$\sigma(\ln y_0) = 0{,}407$; $G(y_0) = 0{,}237$

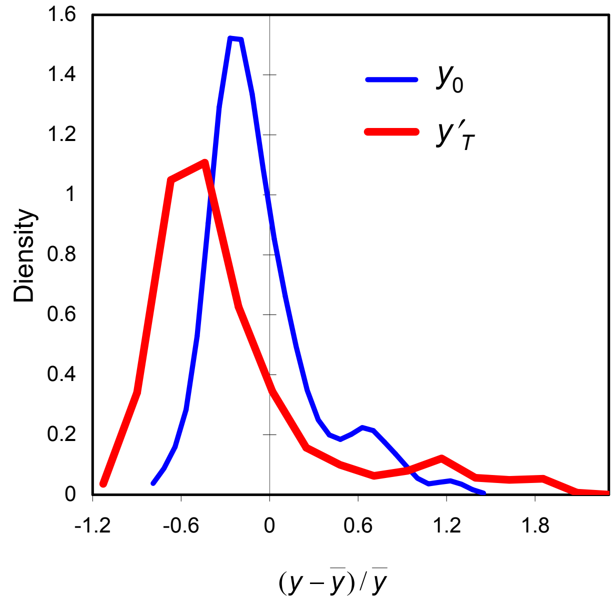
$\{y'_{iT}\}$ = permutation of $\{y_{i0}^2\}$

$\sigma(\ln y'_T) = 0{,}815$, $G(y'_T) = 0{,}503$

Regression	Constant α		β	
	Estimate	p-value	Estimate	p-value
$0 \to T$	11,985 (1,889)	0,000	**–0,375** (0,217)	0,087
$T \to 0$	5,992 (0,945)	0,000	–0,844 (0,054)	0,000

$0 \to T$: $\ln(y'_{iT}/y_{i0}) = \alpha + \beta \ln y_{i0} + \varepsilon_i$

$T \to 0$: $\ln(y_{i0}/y'_{iT}) = \alpha + \beta \ln y'_{iT} + \varepsilon_i$

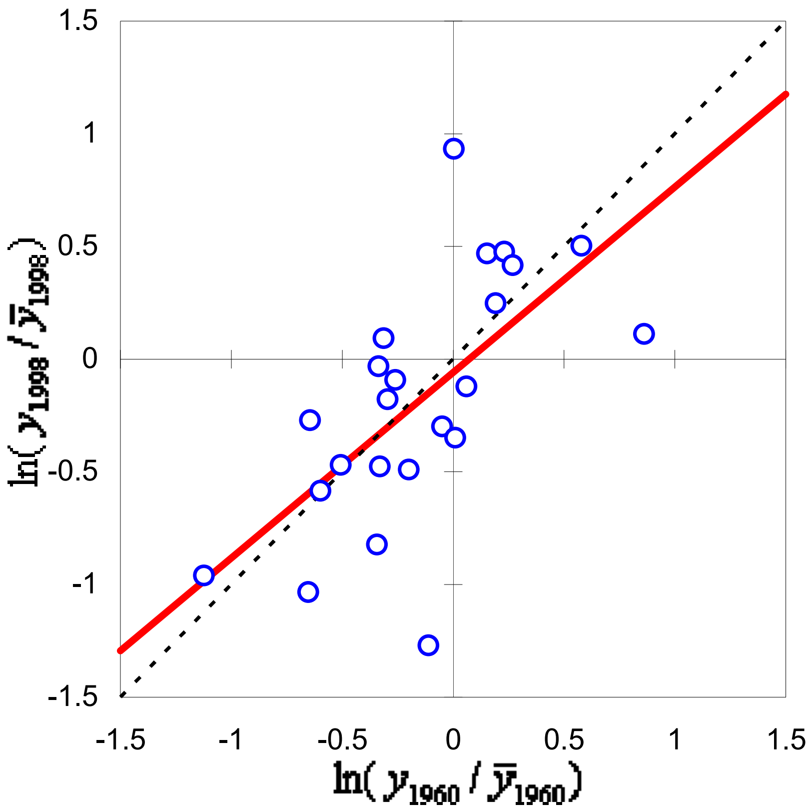


β-convergence when moving from y_0 to y'_T

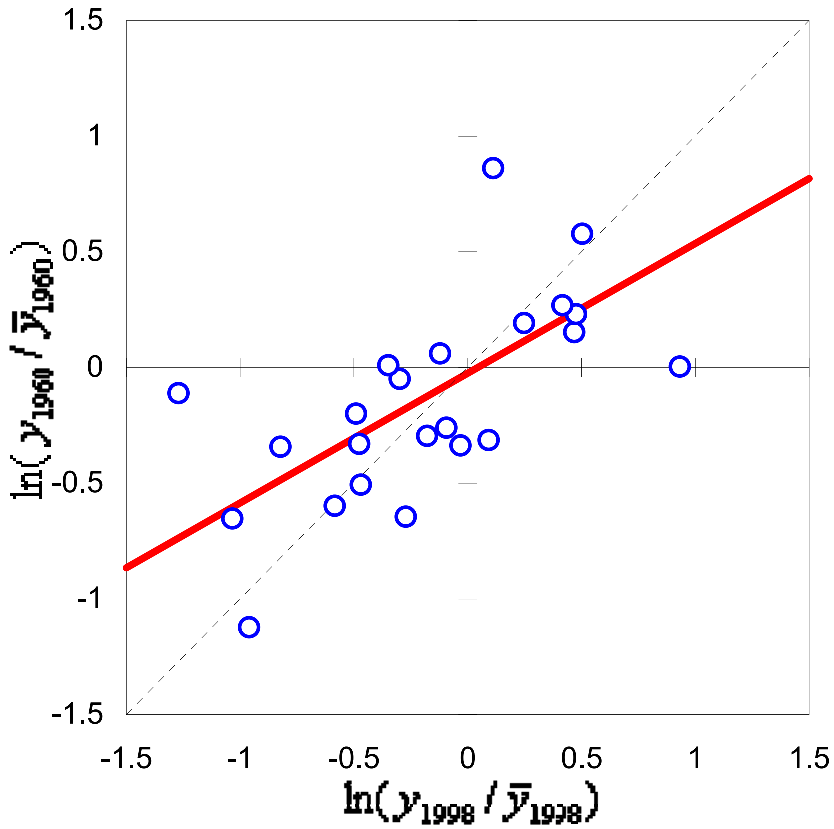
EXAMPLE

Latin America, real GDP per capita in 1960 and 1998

$\sigma_{1960} = 0{,}457, \ \sigma_{1998} = 0{,}554$

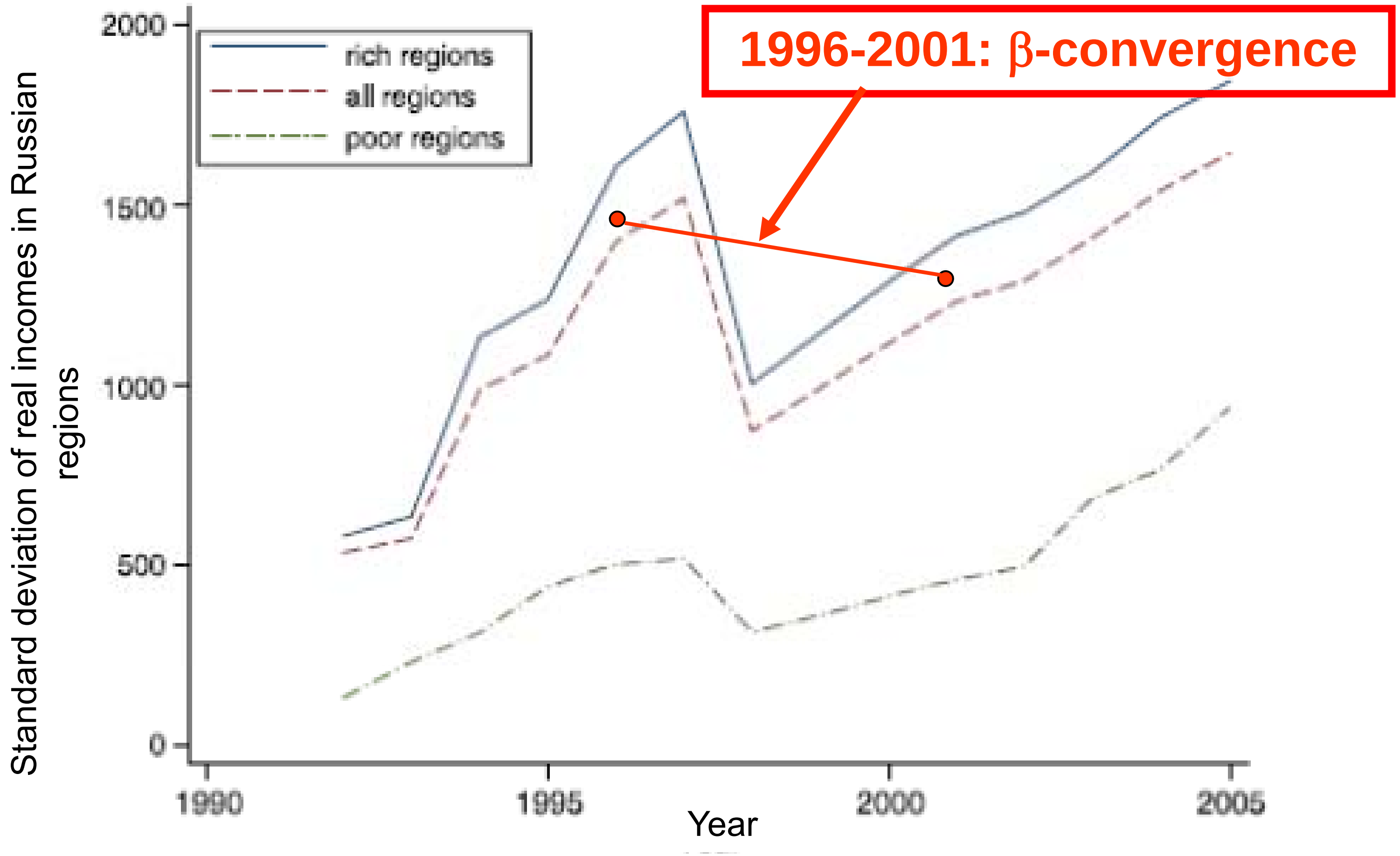


$\ln y_{1998} = 2{,}003 + 0{,}823 \ln y_{1960}$



$\ln y_{1960} = 3{,}179 + 0{,}561 \ln y_{1998}$

LOSS OF INFORMATION ON DYNAMICS



Source: L.Solanko, *Post-Communist Economies*, 2008, Vol. 20, No. 3, p. 291.

CONDITIONAL β-CONVERGENCE

$$\ln(y_{iT}/y_{i0}) = \alpha + \beta \ln y_{i0} + \Sigma\alpha_k \ln x_{ik} + \varepsilon_i$$

$$\ln(y_{iT}/y_{i0}) - \Sigma\alpha_k \ln x_{ik} = \ln(y°_{iT}/y_{i0}) = \alpha + \beta \ln y_{i0} + \varepsilon_i$$

$$\ln(y_{iT}) = \Sigma\alpha_k \ln x_{ik} + \nu_i\,, \ \ln(y°_{iT}) = \hat{\nu}_i$$

$$\boldsymbol{\ln(y°_{iT}/y_{i0}) = \alpha + \beta \ln y_{i0} + \varepsilon_i\,,}$$

The latter is completely similar to the unconditional β-convergence regression. Hence, all the aforesaid regarding unconditional β-convergence is true for conditional convergence.

CONCLUSIONS

Analysis of β-convergence (both unconditional and conditional) **is useless** в in applied researches aiming at indentifying trends of the evolution of spatial income inequality.

This does not imply that the β-convergence concept is fallacious. The matter is not the concept itself, but wrong interpretation, misuse of the concept. By means of the β-convergence analysis, researchers tackle a question which fundamentally cannot be answered by this method.

Its application field is very narrow, namely, testing (verification) of economic growth models. Empirical analysis of β-convergence makes it possible to find out whether the behavior of economies possesses some property emerging from the neoclassical growth model, **and nothing more**.