3. CROSS-SECTION APPROACH

3.2. Spatial econometrics

"CAUSAL ANALYSIS:"
WHAT ARE REASONS FOR INEQUALITY?

$z_i = \alpha_0 + \alpha_1 x_{i1} + \ldots + \alpha_m x_{im} + \varepsilon_i,\ i = 1,\ldots,N$

or, in matrix notations,

$\mathbf{z} = \mathbf{X}\alpha + \varepsilon$

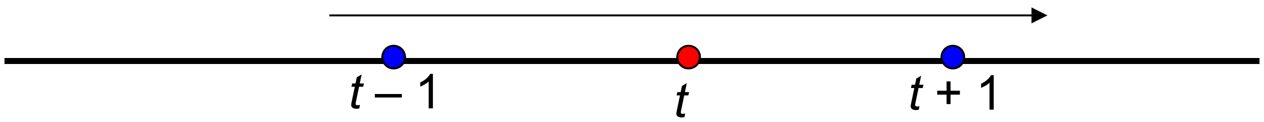
Such equations may look like the "conditional β-convergence" equation, but no special meaning is now attached to the coefficient on y_{i0}. It enjoys the same rights as other variables.

Specificity: Locations may interact with one another in an unknown way, so causing **spatial autocorrelation:** $\mathrm{cov}(z_i, z_j) = \mathrm{E}(z_i z_j) - \mathrm{E}(z_i)\mathrm{E}(z_j) \neq 0$ (for $i \neq j$).

Spatial econometrics is an area of econometrics that deals with spatial autocorrelation.

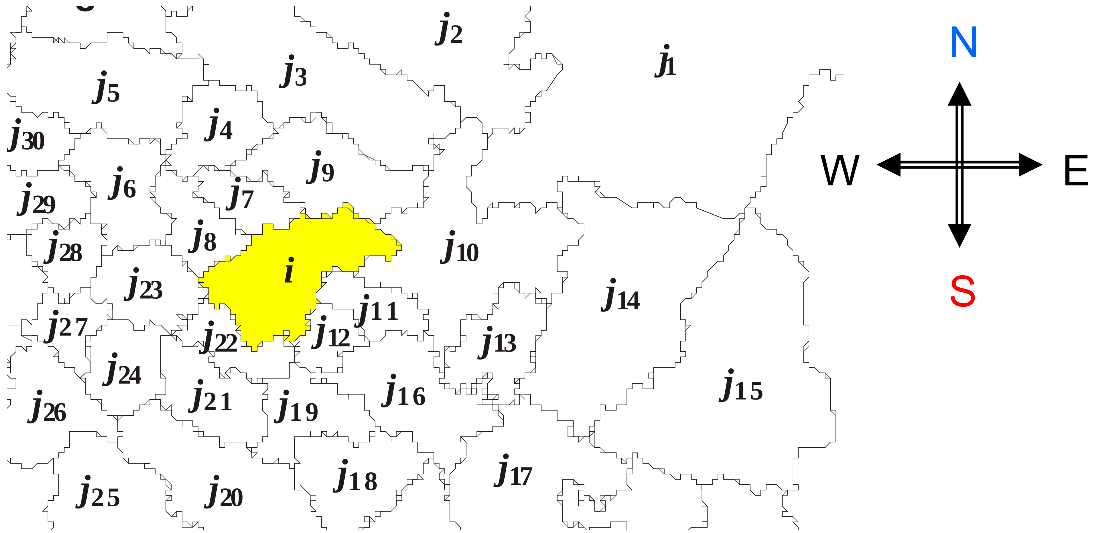
SPATIAL AUTOCORRELATION

Time:



One-dimensional, unidirectional, unique preceding observation

Space:



Two-dimensional, isotropic (no preferential direction).

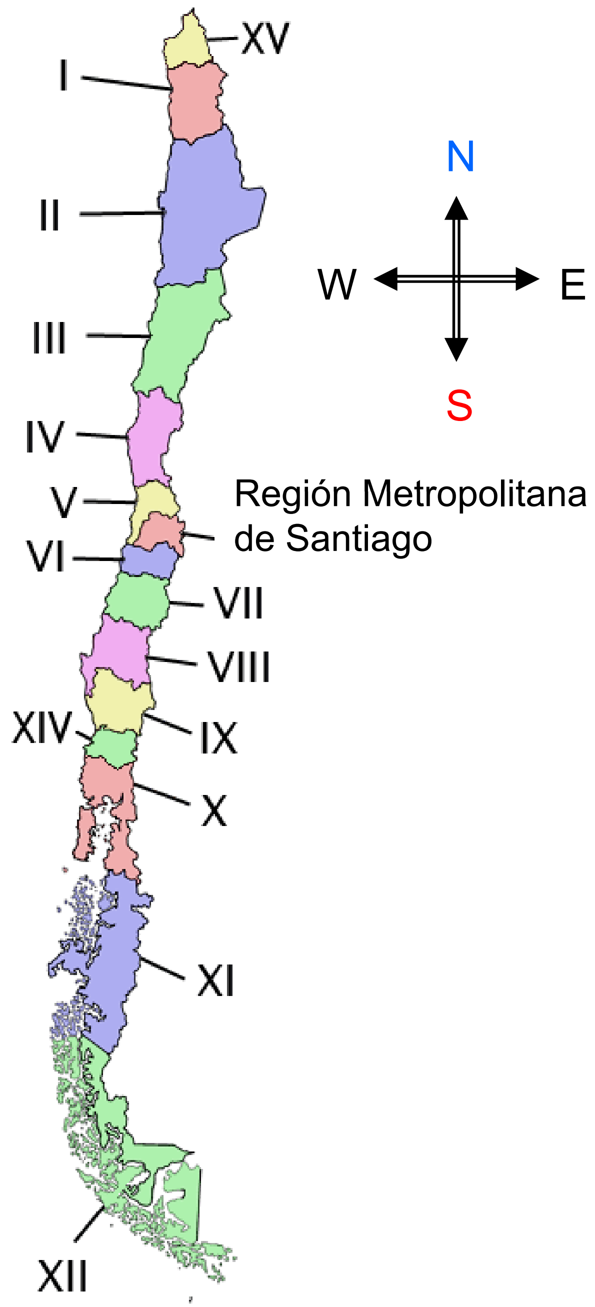
What is preceding observation for *i* (a lag of *i*)?

REGIONS OF CHILE

The first level of the administrative division of Chile provides almost one-dimensional construction.

Nonetheless, what observation is lag for, say, region VII?

VIII or VI?

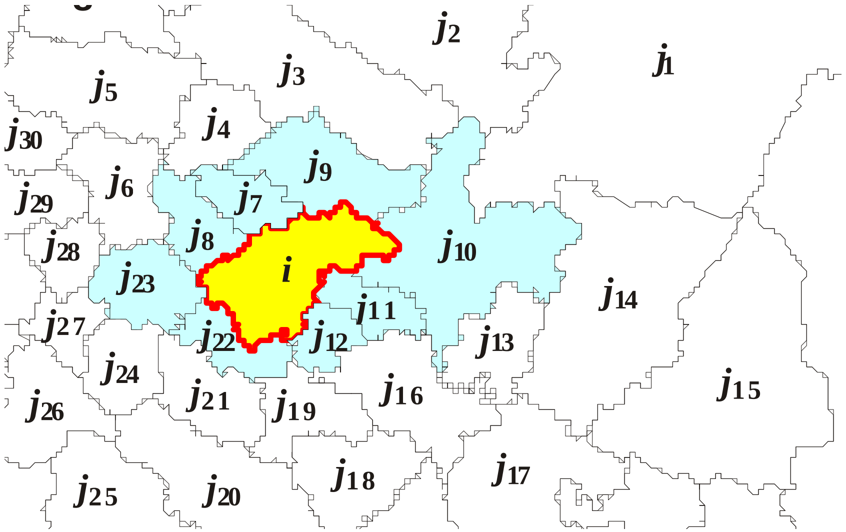


SPATIAL LAG

- $z_{"i-1"} = \sum_{j \in S_i} w_{ij} z_j$, S_i = a set of locations neighboring to *i*
- **W** = (w_{ij}) is the **spatial weight matrix** ($N \times N$)
- In matrix notations, the spatial lag is $\mathbf{z}_{(-1)} = \mathbf{Wz}$
- $\sum_{j=1}^{N} w_{ij} = 1$
- $w_{ii} = 0$ (a location is not a neighbor of itself)

Thus, the spatial lag is a weighted average of the indicator under study over neighboring locations

WHAT ARE NEIGHBORING LOCATIONS?



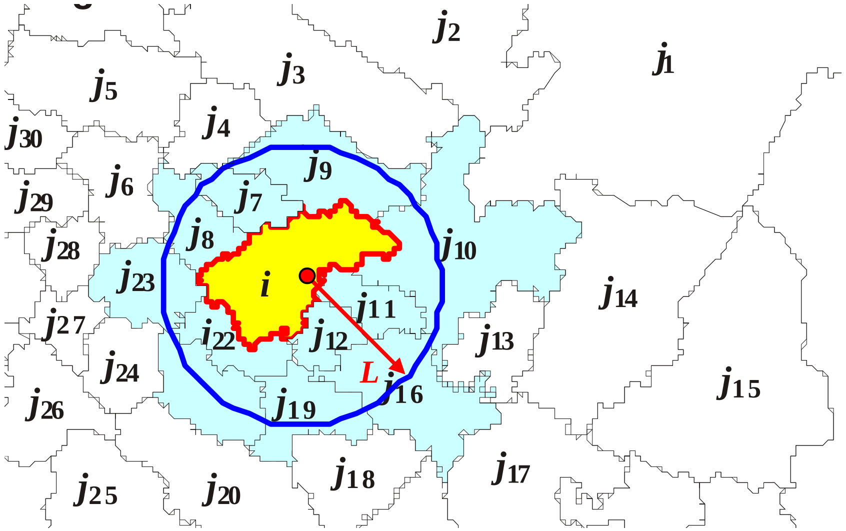
Adjacency:

S_i = {locations that have a common border with *i*}

$w_{ij} = 1/n_i$ if *j* has a common border with *i*, otherwise $w_{ij} = 0$;

n_i = number of locations in S_i

WHAT ARE NEIGHBORING LOCATIONS?



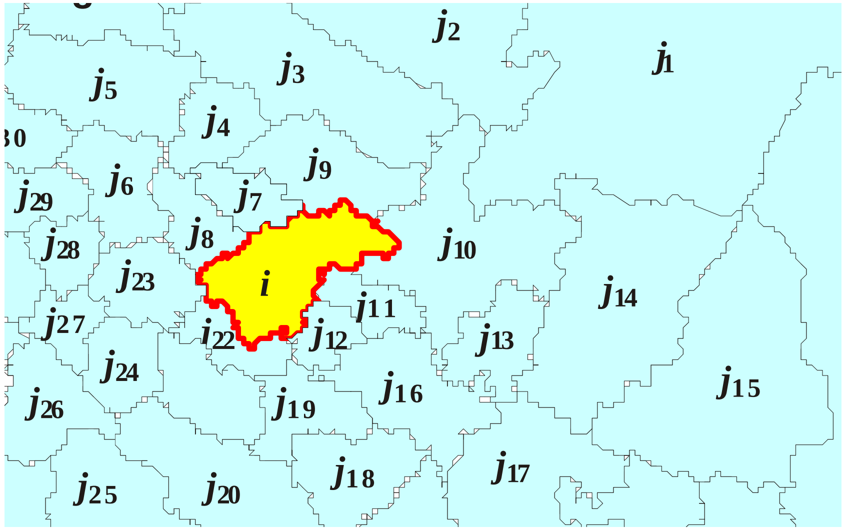
Proximity:

$S_i = \{j \mid L_{ij} \leq L\}$

$w_{ij} = 1/n_i$ if $L_{ij} \leq L$, otherwise $w_{ij} = 0$;

L_{ij} = distance between *i* and *j*; n_i = number of locations in S_i

A DIFFERENT WAY TO CONSTRUCT SPATIAL WEIGHTS



Distance:

$S_i = \{j \neq i\}$

$w_{ij} = f(L_{ij})$, $df/dL < 0$, $f(0) = 0$.

For example, $w_{ij} = c_j/L_{ij}$ or $w_{ij} = c^j/L^2_{ij}$ (with $w_{ii} = 0$);
c_j = normalizing factor.

SPATIAL REGRESSION MODELS

- **Spatial autoregressive model**

  assumes the dependent variable to be autocorrelated:

  $\mathbf{z} = \mathbf{X}\alpha + \rho\mathbf{W}\mathbf{z} + \varepsilon$, $\rho =$ a spatial autoregressive coefficient

- **Spatial error model**

  assumes the residuals to be autocorrelated:

  $\mathbf{z} = \mathbf{X}\alpha + \nu$, $\nu = \rho\mathbf{W}\nu + \varepsilon \quad \Rightarrow$

  $\mathbf{z} = \mathbf{X}\alpha + \rho\mathbf{W}\mathbf{z} + \rho\mathbf{W}\mathbf{X}\alpha + \varepsilon$

The spatial lag term **Wz** is correlated with the disturbances!

DIRECT REPRESENTATION OF SPATIAL AUTOCORRELATION

- Covariance matrix $\boldsymbol{\Omega} = (\omega_{ij}) = (\mathrm{cov}(\varepsilon_i, \varepsilon_j))$ is modified rather than the regression $\mathbf{z} = \mathbf{X}\alpha + \varepsilon$ (*) itself.
- Assumptions as to the structure of Ω are needed, e.g. $\omega_{ij} = \sigma^2 f(L_{ij})$.
- Example: $\omega_{ij} = \kappa + \mu\exp(-\lambda L_{ij})$, $\lambda > 0$; to estimate, run $\hat{\varepsilon}_i \hat{\varepsilon}_j = \kappa + \mu\exp(-\lambda L_{ij})$, where $\{\hat{\varepsilon}_r\}$ are residuals of (*). Thereafter reesimate (*) with the use of the modified Ω.

TESTING FOR SPATIAL AUTOCORRELATION

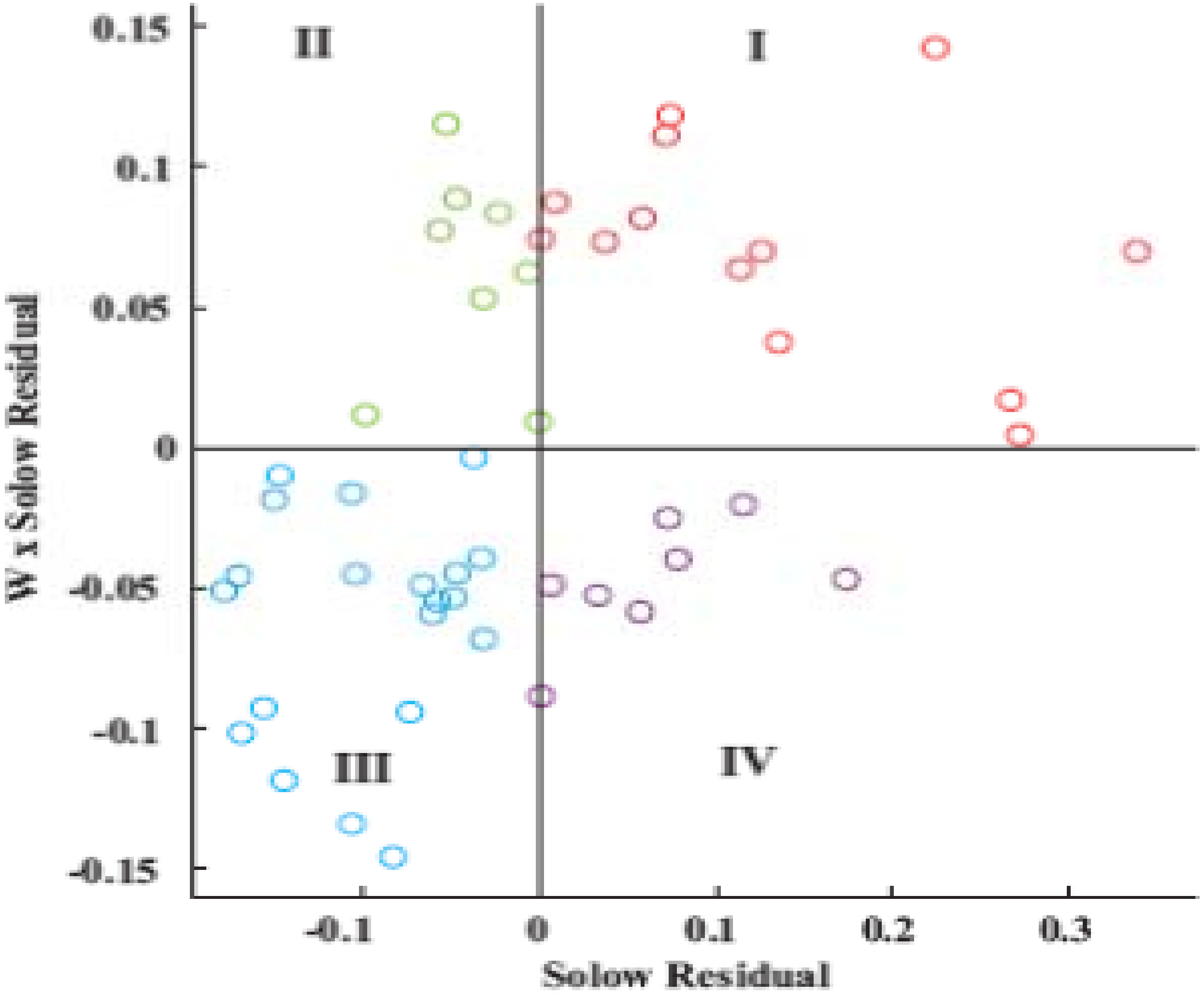
- Moran I statistic

$$I = \frac{\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}(z_i - \overline{z})(z_j - \overline{z})}{\frac{1}{N}\sum_{i=1}^{N}(z_i - \overline{z})^2 \sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}}$$

or $I = \mathbf{z'Wz/z'z} = \mathrm{cov}(z, z_{\mathbf{(-1)}})/\sigma^2$

Other statistics: Geary's C, Ord and Getis' statistic, etc.

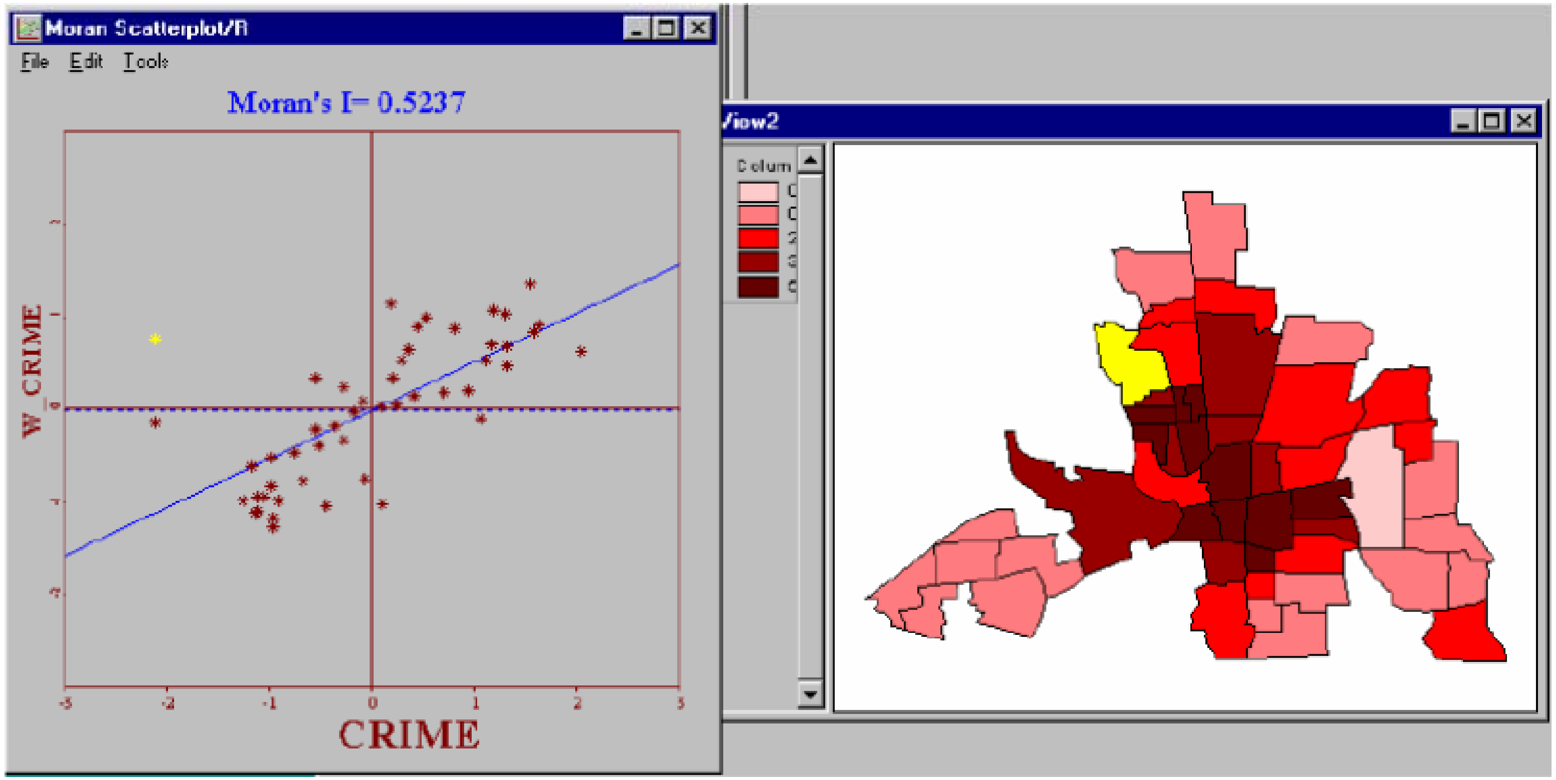
VISUALIZATION: MORAN SCATTER PLOT



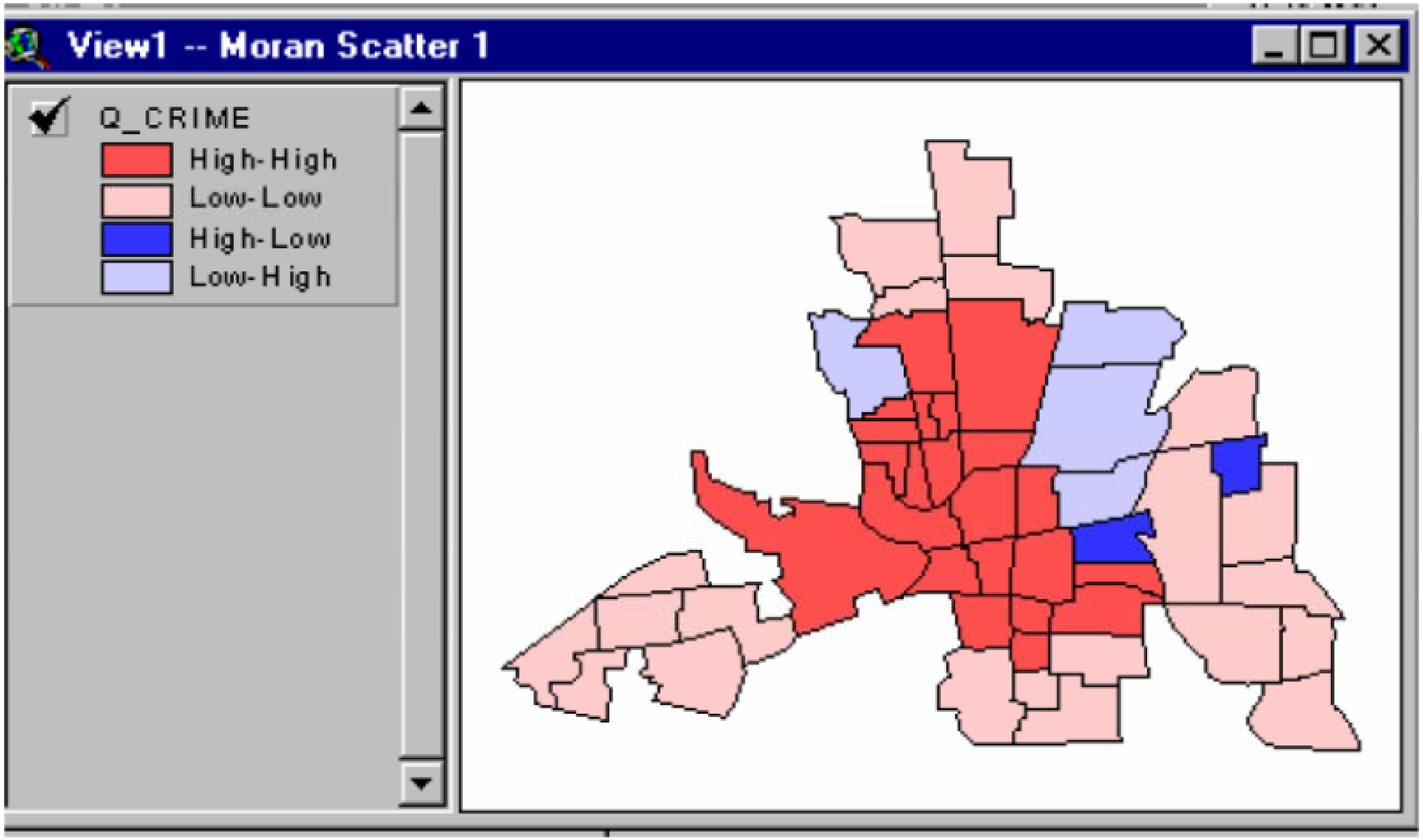
LeSage J., Pace R.K. *Introduction to Spatial Econometrics*, 2009,
Fig. 1.4. Moran scatter plot of 2001 US states factor productivity

VISUALIZATION: MORAN SCATTER PLOT MAP

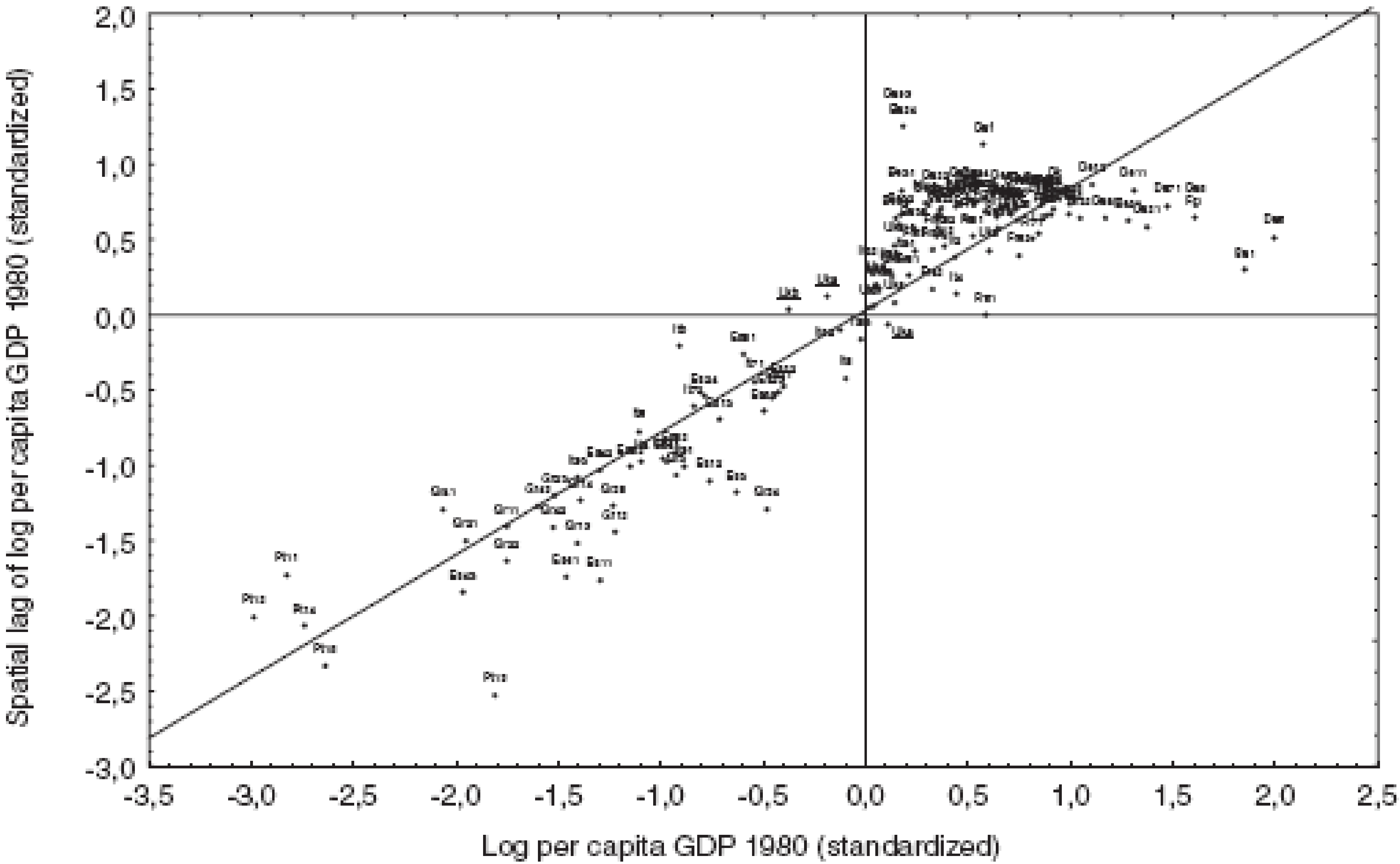
LeSage J., Pace R.K. *Introduction to Spatial Econometrics*, 2009,
Fig. 1.5. Moran plot map of US states 2001 factor productivity



Moran Scatterplot for Columbus crime
scatterplot of crime against spatial lag of crime (w_crime)
standardized values



Moran Scatterplot Map for Columbus crime
four quadrants of the scatterplot



Ertur C., Le Gallo J., Baumont C. The European regional convergence process, 1980-1995: do spatial regimes and spatial dependence matter? *International Regional Science Review*, 2006, 29 (1), 3-34.