4. TIME SERIES APPROACH

Time series of an indicator y over $t = 0,\ldots,T$ across locations $i = 1,\ldots,N$:

$$\{y_{it}\}_{t = 0,\ldots,T;\; i = 1,\ldots,N}$$

Do $\boldsymbol{y}_i$ become closer (converge) to one another over time?

- Pairwise: $y_{ijt} = y_{it} - y_{jt}$ for all $N(N - 1)/2$ pairs (i, j)

  Disadvantage: cumbersome
- Benchmark: y_{ijt} with fixed j ($N - 1$ pairs)

  Disadvantage: intransitivity

  ($y_{it} \to y_{jt}$ and $y_{kt} \to y_{jt}$ does not imply $y_{it} \to y_{kt}$)
- Comparison with the mean: $y_{ijt} = y_{it} - \bar{y}_t$

  Disadvantage: some series in the mean may be non-stationary
- Panel analysis: $\{y_{it}\}$ with location and time fixed effects

  Disadvantage: common coefficients for all series

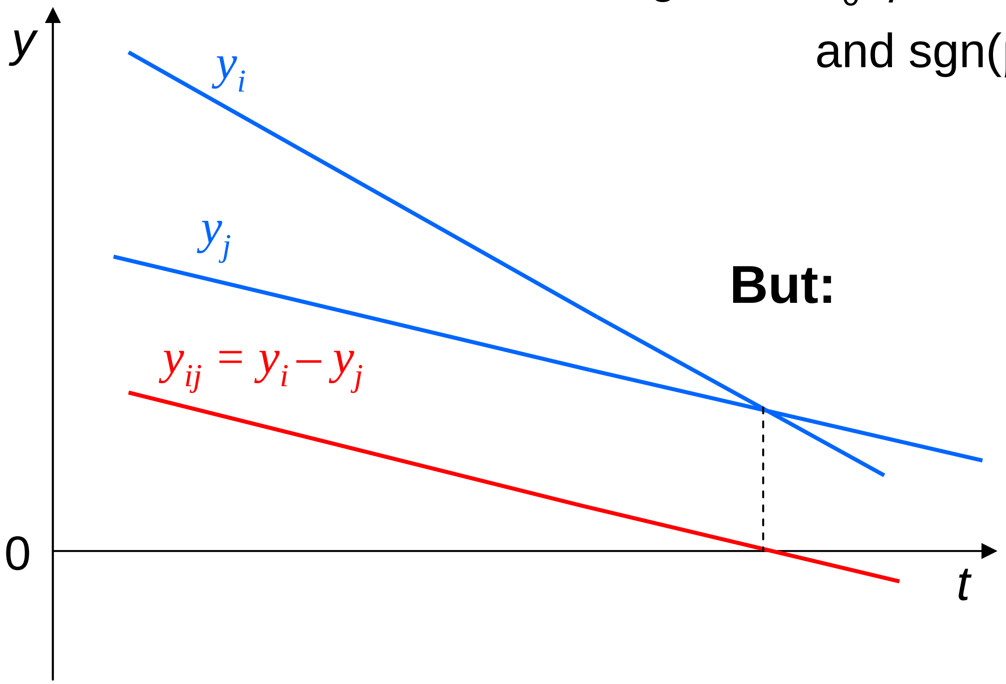
LINEAR TREND

$$y_{it} - y_{jt} \equiv y_{ijt} = \alpha + \beta t + \nu_t$$

$$\nu_t = \rho \nu_{t-1} + \varepsilon_t$$

Convergence: H_0: $\rho < 1$ and $\beta \neq 0$;

and $\text{sgn}(\beta) = -\text{sgn}(y_{ij0})$



But:

STRICT DEFINITION OF CONVERGENCE

Bernard A.B., Durlauf S.N. Convergence in international output. *Journal of Applied Econometrics*, 1995, V. 10, No. 2:

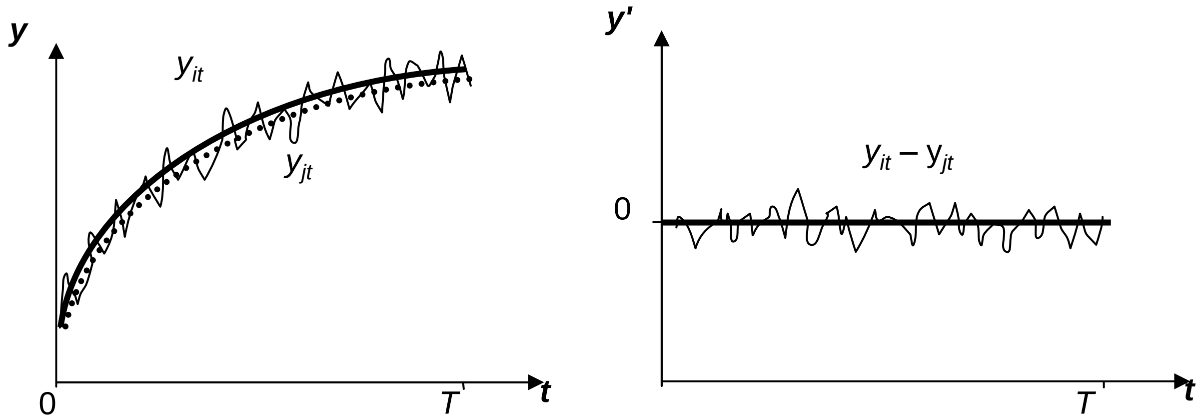
$$\lim_{t \to \infty} E(y_{it} - y_{jt}) = 0$$

However, they test $y_{it} - y_{jt}$ for stationarity:

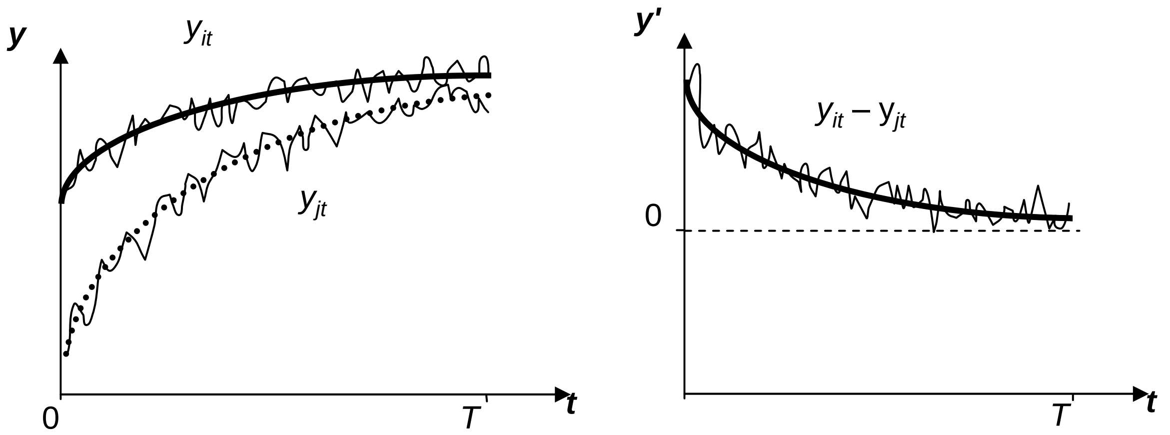
$y_{ijt} = \nu_t$, $\nu_t = \rho\nu_{t-1} + \varepsilon_t$

Half-life time: $\log(0.5/\rho)$

SHORT-RUN VS. LONG-RUN CONVERGENCE



Short-run convergence (ordinary cointegration)



Long-run convergence (catching-up) combined with short-run one

AN ALTERNATIVE TEST FOR CONVERGENCE:

Nahar S., Inder B. Testing convergence in economic growth for OECD countries. *Applied Economics*, 2002, V. 34, No. 16

$y_{ijt}^2 = h(t) + \varepsilon_t = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \ldots + \alpha_k t^k + \varepsilon_t$

If long-run convergence happens, $dh(t)/dt < 0$ must hold for all t.

Then $$\frac{1}{T}\sum_{t=0}^{T}\frac{dh(t)}{dt} = \sum_{i=1}^{k}\alpha_i \frac{i}{T}\sum_{t=0}^{T} t^{i-1} < 0.$$

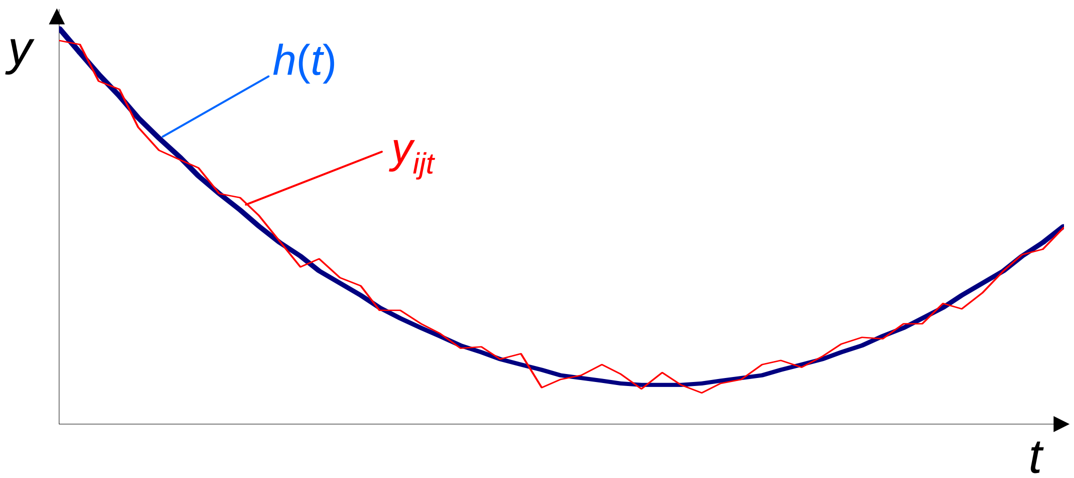
However, $$\frac{1}{T}\int_0^T \frac{dh(t)}{dt}dt = \frac{1}{T}(h(T) - h(0)) < 0.$$

(ARTIFICIAL) EXAMPLE

$y_{ijt}^2 = 100 - 6t + 0.1t^2 + \varepsilon_t$, $t = 0,\ldots,50$

$$\frac{1}{T}\sum_{t=0}^{T}\frac{dh(t)}{dt} = \alpha_1 + \alpha_2\frac{2}{T}\sum_{t=0}^{T} t \;=\; -6 + 2\cdot 0.1\cdot 1275/51 = -1 < 0$$

$(h(T) - h(0))/T = (50 - 100)/50 = -1 < 0$



A DIFFERENT WAY

Disadvantages of the Nahar-Inder method:

(a) overly general representation of the long-run trend;

(b) no autocorrelation.

Ways out: take as asymptotically decaying function as h(t) and take account of autocorrelation.

$y_{ijt} = h(t) + \nu_t,\ \nu_t = \rho\nu_{t-1} + \varepsilon_t \quad \Rightarrow$

$\Delta y_{ijt} = \lambda y_{ij,t\text{-}1} + h(t) - (\lambda + 1)h(t-1) + \varepsilon_t \quad (\lambda = \rho - 1)$

$h(t)$: $\ln(1 + \gamma e^{\delta t}),\ \delta < 0;$

$\gamma e^{\delta t},\ \delta < 0;$

$\gamma/(1 + \delta t),\ \delta > 0;$

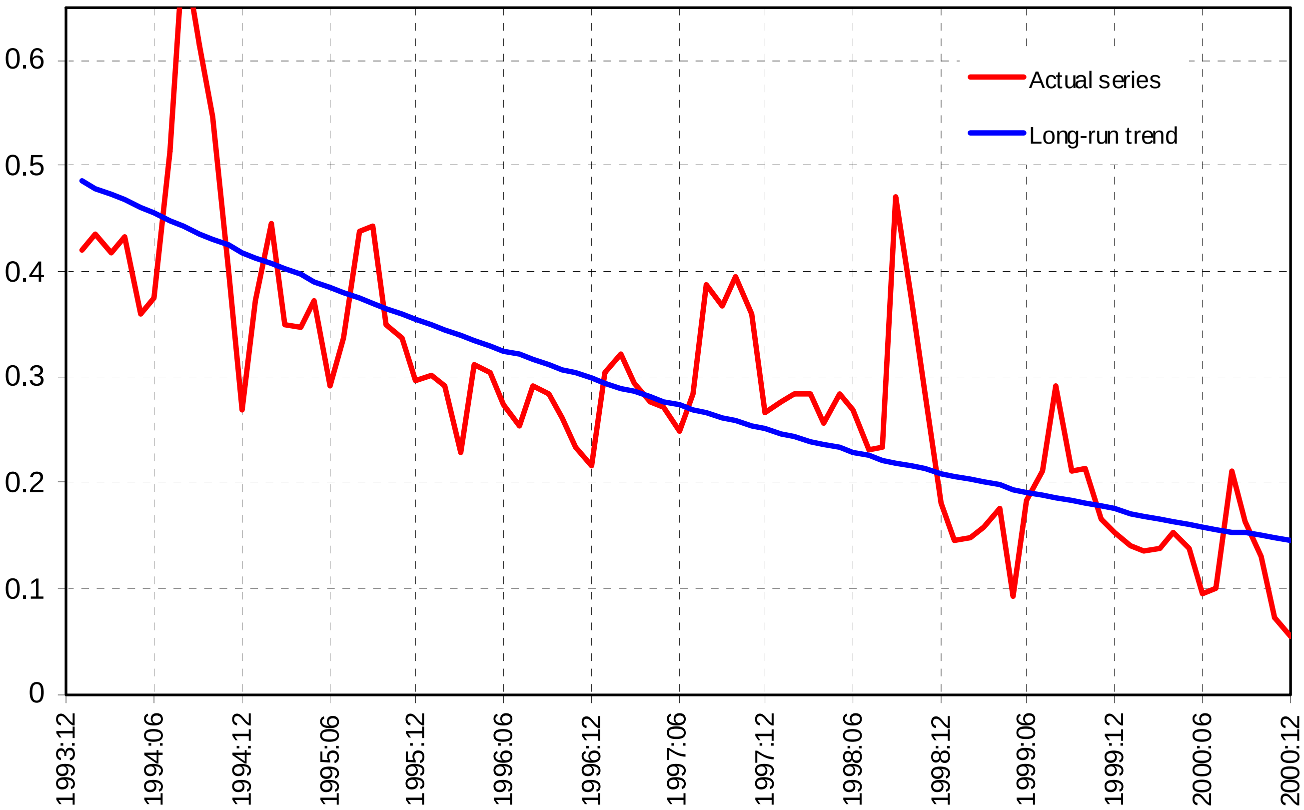
etc.

EXAMPE 1

y_{ijt} = log(p_{it}/p_{jt}), p = price of the staples basket

i = the Arkhangelsk Oblast, j = the Saratov Oblast

$h(t) = \ln(1 + \gamma e^{\delta t})$



EXAMPE 2 (with structural break)

$y_{ijt} = \log(p_{it}/p_{jt})$, p = price of the staples basket
i = the Republic of Komi, j = the Saratov Oblast
$h(t) = \ln(1 + \gamma e^{\delta t})$